CMM-SolMech 2022 conference

S01 Advanced discretization methods

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VIRTUAL ELEMENT METHOD IN 2D ELASTICITY PROBLEM IN CONTACT MECHANICS

Paweł Goliszewski[1]

[1] Faculty of Mathematics and Computer Science, Jagiellonian University, Poland

pawel.goliszewski@doctoral.uj.edu.pl

We tackle a problem of calculating deformation and stress for homogenous, isotropic and elastic body described by generalized Hooke's law. Contact boundary conditions, like bilateral contact with friction, are used. Weak formulation of this problem leads us to variational inequalities which can be solved using Virtual Element Method (VEM).

VEM is one of the generalisations of Finite Element Method (FEM). Compared to FEM, VEM has fewer limitations in choosing elements which make up the grid. The elements can be polygons of any number of sides. The downside of the method is the final solution is known only at the boundaries of elements.

The presentation contains the description of the model, a weak formulation of the problem, the idea of VEM, a short mention of an error estimation and numerical results.

AUTOMATIC GENERATION OF INTERNAL POINTS AND CELLS IN THE FAST MULTIPOLE BOUNDARY ELEMENT METHOD

Barbara Ciszyńska[1], **Jacek Ptaszny**[1]

[1] Department of Computational Mechanics and Engineering, Silesian University of Technology, Poland

barbara.ciszynska@polsl.pl

The use of the boundary element method (BEM) in combination with the fast multipole method (FMM) for the analysis of a number of problems of practical importance allowed one to increase the scope of its applications. The main advantage of the fast multipole boundary element method (FMBEM) is the possibility to analyze structures with a large number of degrees of freedom due to its reduced time and memory complexity. For problems that require consideration of volume potentials, in the literature it has been proved that the domain integration approach using the multipole method, despite the need to discretize the inner region, can be more efficient in terms of computation time and accuracy than other BEM formulations.

FMBEM uses a process of grouping boundary elements and inner cells into areas that form a hierarchical structure, stored as a tree. In this work, a method was developed that allows the creation of internal cells in the form of cubes defined by vertices and midpoints for the analysis of three-dimensional problems using boundary elements with quadratic shape functions. They are created due to the defined number of levels of tree, number of elements in cell, and size of internal cells. Each node of the tree can have up to eight successors. This results in a structure called an octree. Internal cells are created by comparing the position of their corner nodes with the nearest boundary nodes and their outward normal vectors. In this way, the need for the user to discretize the inner region has been eliminated. In the preliminary research, the nodes of the cells were applied to calculate the internal quantities in the post-processing stage.

Next, it is planned to develop an efficient method for calculating the volume integrals occurring in the boundary integral equation describing selected deformable structures and to combine it with the FMBEM, which uses quadratic boundary elements. The prepared method would enable the analysis of many issues in the field of mechanics of practical importance such as: analysis of structures loaded with volume forces, gradient materials, or nonlinear problems.

The scientific research was funded from the statutory subsidy of the Faculty of Mechanical Engineering, Silesian University of Technology, Poland.

THE MULTISCALE FINITE ELEMENT METHOD FOR SANDWICH BEAMS AND PLATES

Mateusz Dryzek[1], **Witold Cecot**[1]

[1] Faculty of Civil Engineering, Cracow University of Technology, Poland

mateusz.dryzek@pk.edu.pl

The multiscale finite element method (MsFEM) is a numerical method used to reduce significantly the number of degrees of freedom in numerical simulation of partial differential equations with fast varying coefficients e.g. in modeling of heterogeneous materials. It requires neither the assumption of the scale separation nor the periodicity of the microstructure. Furthermore, the calculations can be easily parallelized, since the MsFEM shape functions that capture the microscale details are computed independently in each coarse-mesh element.

We adapted the method to the analysis of sandwich beams and plates with complex lattice layers using an anisotropic higher-order coarse-scale approximation and novel shape functions that take into account the microscale boundary conditions [1]. Selected numerical examples confirmed the possibility of a reduction of the number of degrees of freedom by even four orders of magnitude.

Such an upscaling was validated by comparison of the computational results with experimental measurements of static bending and free vibrations of 3D printed ABS beams with various print orientations and microstructures, assuming an orthotropic constitutive model of the base material. The numerical and experimental results coincide very well with a number of examples. Moreover, our study revealed that the hierarchical higher-order shape functions can accurately represent free vibrations even for the high frequencies.

This research was supported by the project nr UMO-2017/25/B/ST8/02752 funded by the National Science Centre.

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AN ISOGEOMETRIC MORTAR METHOD WITH OPTIMAL CONVERGENCE AND REDUCED SUPPORT

Wolfgang Dornisch[1], Joachim Stöckler[2]

[1] Chair of Structural Analysis and Dynamics, Brandenburg University of Technology Cottbus-Senftenberg, Germany
[2] Institut für Angewandte Mathematik, TU Dortmund University, Germany

wolfgang.dornisch@b-tu.de

The main feature of isogeometric analysis is the usage of a coherent geometry description for design and analysis. In most cases, Non-Uniform Rational B-splines (NURBS) are used within the frame of the finite element method, which combines the most common geometrical modeling concept with the most common analysis method in structural mechanics. NURBS are a versatile tool for geometric modeling, and in order to define complex geometric structures, a multitude of tensor-product NURBS patches is required. Typical software in Computer-Aided design can manage smoothness requirements across common interfaces of neighboring patches. For the finite element method, a different type of coupling across interfaces must be achieved. Due to the nature of tensor product NURBS, the simple concept of mutual refinement and subsequent coupling by shared degrees of freedom is prohibitively costly and, in some cases, even not possible. A multitude of coupling methods has been proposed over the last years. The most common concepts are known as mortar methods. In particular, the dual mortar method has been shown to yield very efficient computations. A recent paper by the authors has provided an isogeometric mortar method with mathematically proven optimal convergence of the stress errors over the entire domain. We use dual basis functions, which have support only on one interface and avoid interrelations between different interfaces. Models with a large number of intersecting interfaces can be handled. However, the basis functions have full support on the interfaces. In our current contribution, we propose the use of approximate dual basis functions with the advantage of having local support on the interfaces. These functions fulfill the duality only in an approximate way, but still guarantee the optimal degree for the convergence of the mortar method. Since the duality is not fulfilled, an additional lumping of the mortar matrix is introduced. The error of this lumping can be analyzed mathematically and is not significant in comparison to the global approximation error of the finite element method. The use of the approximate dual basis functions restores the local support of basis functions along the interface while the convergence properties remain intact. Numerical examples show the convergence behavior for simple and complex models.

APPLICATION OF RESIDUAL METHODS OF ERROR ESTIMATION TO COUPLED PROBLEMS OF PIEZOELECTRICITY

Grzegorz Zboiński[1]

[1] Institute of Fluid Flow Machinery and University of Warmia and Mazury, Poland

zboi@imp.gda.pl

This paper concerns error estimation and adaptivity control in the hp-adaptive finite element methods. It addresses theoretical and numerical aspects of application of the equilibrated residual method (ERM) to the coupled problems as exemplified by the stationary problem of piezoelectricity. The main difficulty in such an application consists in the lack of an upper bound error property of the ERM global estimator in the case of the coupled problems including piezoelectricity. This lack is due only to stationary nature of the of the weak (variational) formulation for the stationary coupled problems. In the case of piezoelectric problems, the coupled electromechanical potential energy is not minimized but is stationary. Note that the upper bound error property holds for the component uncoupled problems of elasticity (elastostatics) and dielectricity (electrostatics). In both uncoupled problems, minimization of the mechanical and electrical potential energies occurs in the weak formulations of both problems.

The paper addresses the analytical consequences of the lack of the minimization in the context of the three component parts of the electromechanical potential energy: mechanical, electric and coupling ones. Also, the consequences of this lack for the equations of the equilibrated residual method are discussed, including the lack of the upper bound error property for the piezoelectric problems. In this context, the possible definitions of the global error and the global error estimator/indicator and their components are proposed. Next, the applicability of the method to error assessment and adaptivity control are discussed.

In the numerical part of the paper, we try to give an answer to the question how to use the equilibrated residual method effectively, in the case of lack of the mentioned upper bound error property of the estimator/indicator. Two aspects are considered, the effective error estimation and the effective adaptivity control. We start this part of the paper with the formulation of the ERM local (element) problems for the piezoelectricity case and for the cases of elasticity and dielectricity as well. The properties of the mechanical, electrical and coupling parts of the error indicator are demonstrated, and of their sum as well. Effectivity indices corresponding to these parts and to their sum are calculated and discussed. Some comparisons of the error measures, error indictors and the effectivity of the latter to the uncoupled problems of elasticity and dielectricity are performed. Also, the control of the hp-adaptivity with the applied error indicators is presented in the case of piezoelectricity and compared to the analogous problems of elasticity and dielectricity.

NUMERICAL ANALYSIS OF A NON-CLAMPED DYNAMIC THERMOVISCOELASTIC CONTACT PROBLEM

Krzysztof Bartosz[1], Pawel Szafraniec[1], Michal Jureczka[1], Piotr Bartman[1]

[1] Faculty of Mathematics and Computer Science, Jagiellonian University, Poland

piotr.bartman@doctoral.uj.edu.pl

In this work we deal with a dynamic contact of a viscoelastic body with a foundation involving thermal effect. The contact is frictional and the friction law is modeled by a nonmonotone relation between the tangential velocity and the tangential stress. The nonmonotone character of friction law introduces one of main challenges in the modeling of contact mechanical problems. We provide the numerical analysis of the problem based on the fully discrete scheme. The main result consists in the estimate of the error between the discrete and the exact solution. We show that the error estimate is linear with respect to the temporal and the spatial discretization parameters in the case when the spatial discretization is based on the first order Finite Element Method. We also carry out the computer simulations using the 'conmech' - original software package for error estimation in Contact Mechanics. Simulations illustrate and validate the linear rate of convergence with respect to the discretization parameters.

PARALLEL COMPUTATIONS WITH THE SIMPLEX-SHAPED SPACE-TIME FINITE ELEMENT METHOD IN STRUCTURAL DYNAMIC

Czesław Bajer[1], Bartłomiej Dyniewicz[1], Mateusz Bajkowski[2]

[1] Institute of Fundamental Technological Research, Poland
[2] Faculty of Production Engineering, Warsaw University of Technology, Poland

cbajer@ippt.pan.pl

An efficient algorithm for parallel computing for large scale structural vibration problems is proposed. The presented original direct method uses simplex-shaped space-time finite elements that enable a direct decoupling of the variables while assembling global matrices. The method uses the consistent stiffness, inertia and damping matrices, and can operate with non-symmetric matrices. The computational cost is independent of the bandwidth of the matrix in the classical meaning since only non-zero coefficients are retained. The algorithm was successfully implemented. The computational gain increases with the number of nodes and the dimensionality of the problem and can reach thousands of times even in the case of moderate discrete meshes. A large size 3-D example and comparison prove the efficiency of this space-time approach.

The computational time is significantly shorter than for other methods. The well known commercial application ANSYS requires a time proportional to the number of degrees of freedom in the analysed structure. The proposed space-time simplex element method only requires a time independent of the number of degrees of freedom since the distributed solution is carried on in parallel threads. The size of the problem that can be solved during a single computational stage depends on the number of nodes and blocks on the GPU card. The further technical evolution of GPU card architecture together with the CUDA environment will allow both increasing the number of nodes treated in parallel and decreasing the time of a single computational step.

Graphic adapters should be designed taking into account the elaborated algorithms for particular real-time dynamic simulations. Also, special algorithms should be developed for matrix computations. Iterative methods are not so promising here as direct methods are. Generally, the GPU card used with the space-time finite element method is even more efficient for 3-D tasks with complex meshes compared with classical computational methods than it is for 2-D tasks. The proposed efficient solver for various physical problems is universal and can be successfully applied to simulations, although more detailed further analysis of the leading idea could significantly improve the proposed computer procedure.

Acknowledgment This research has been supported within the projects UMO-2017/26/E/ST8/00532 and UMO-2019/33/B/ST8/02686 funded by the Polish National Science Centre, which is gratefully acknowledged by the authors.

A FEM FOR COMPUTING DISPLACEMENT AND PRESSURE INDEPENDENTLY IN LINEAR INCOMPRESSIBLE ELASTICITY

Adam Zdunek[1], Waldemar Rachowicz[2], Michael Neunteufel[3]

[1] Ber-Rit HB, Sweden
[2] Computer Science and Telecommunications, Cracow University of Technology, Poland
[3] Institute for Analysis and Scientific Computing, Technische Universitat Wien, Austria

wrachowicz@pk.edu.pl

We investigate a possibility of determination of the displacement u(x) independently from the pressure reaction p(x) in variational boundary-value problems of incompressible linear elasticity with various boundary conditions. A novel three step finite element procedure is used. The main features of the presented approach are: usage of the superposition principle, an L2-orthogonal Helmholtz decomposition of the body force into potential and solenoidal parts, and assuming a direct sum decomposition of the displacement into a divergence-free part and its orthogonal complement.

Our implementations use standard H1 or H(curl) shape functions for u(x) and L2 or H1 functions for p(x). A common belief is that p(x) is equipollent to a potential body force (Leonhard Euler, 1756). Our numerical experiments however, show that a divergence-free body force may cause a non-trivial pressure p(x) in the case of fully clamped boundary conditions u(x)=0, preventing an independent determination of u(x) and p(x). Relaxing to a mixed boundary condition either with the normal trace n.u=0 or the tangential trace n x u=0 allows us to compute u(x) and p(x) independently. This is not necessarily equivalent to computing u(x) pressure robustly in the sense of the work of Lederer et al. (P. L. Lederer, A. Linke, C. Merdon and J. Schöberl, Divergence-free Reconstruction Operators for Pressure-Robust Stokes Discretizations With Continuous Pressure Finite Elements, SIAM Journal on Numerical Analysis, 55 (3), 1291-1314, 2017).

Our findings enrich the field of investigations considering pressure robustness by invoking divergence free body forces. In analogy with Stokes problems the relevance of this research concerns incompressible linear elastic materials. The Neumann part of the mixed boundary condition will be discussed. Extension to problems with non-homogeneous essential boundary conditions and natural boundary conditions are under consideration.

AN HP-ADAPTIVE DISCONTINUOUS PETROV-GALERKIN FINITE ELEMENT METHOD FOR COMPRESSIBLE VISCOUS FLOWS

Waldemar Rachowicz[1], **Witold Cecot**[2]

[1] Computer Science and Telecommunications, Cracow University of Technology, Poland
[2] Civil Engineering, Cracow University of Technology, Poland

witold.cecot@pk.edu.pl

A Discontinuous Petrov-Galerkin (DPG) technique is useful for approximation of boundary-value problems with small perturbation parameters as it allows one for a robust procedure of solving such problems with the Finite Element Method (FEM). We recently extended the 2D version of DPG method for compressible viscous flows due to Chan et al. (J. Chan, L. Demkowicz and R. Moser, A DPG method for steady viscous compressible flow, Computers and Fluids, 98, 69-90, 2014) to three dimensions in W. Rachowicz, A. Zdunek and W. Cecot, A discontinuous Petrov-Galerkin method for compressible Navier-Stokes equations in three dimensions, Computers and Mathematics with applications, 102, pp. 113-136, 2021.

Both procedures exploit h-adaptivity, including its anisotropic version, to resolve the typical irregularities of solution to supersonic viscous flow problems: shocks and boundary layers. The present work is focused on application of higher-order elements to approximate highly irregular but analytic solutions in boundary layers. Such an approximation is known to be essentially more effective in this situation than interpolation with larger number of finer elements of low order. Yet, higher order elements are not often used in the context of many schemes as they frequently lead to loss of stability. In contrary, stable behavior of the DPG method is insensitive to the kind of approximation being used. The price that is paid for the discrete stability of DPG is the essential increase of the computational cost, if compared to other popular techniques.

We make an attempt to equilibrate this cost by limiting the higher order approximation to the direction perpendicular to the boundary layer, as the solution varies smoothly along the solid wall except the area of stagnation point or shock-boundary layer interaction, where approximation is isotropic. We also use fast integration of stiffness matrices of higher order elements and a two-grid iterative solver. Adequate control of its accuracy of convergence and the number of iterations brings additional savings in the CPU time. Both, the element computations and the iterative solver are implemented to use parallel processing.

MODELLING OF THIN METAL FILM HEATING USING THE DPL EQUATION WITH TEMPERATURE-DEPENDENT PARAMETERS

Ewa Majchrzak[1], **Bohdan Mochnacki**[2]

[1] Silesian University of Technology, Poland
[2] University of Occupational Safety Management in Katowice, Poland

ewa.majchrzak@polsl.pl

Thermal processes in domain of thin metal film subjected to a laser pulse are considered. The heating/cooling process is described by the dual phase lag equation (DPLE) with the temperature-dependent thermophysical parameters (volumetric specific heat and thermal conductivity) and the appropriate boundary-initial conditions. From the mathematical and numerical points of view, the model is essentially more complicated compared to the linear task (e.g. [1, 2]). Taking into account the geometrical properties of the laser beam, the problem is considered as an axially-symmetrical one, and the conventionally assumed radius of metal film sub-domain enables to assume the adiabatic conditions on the outer boundaries of the area.

At the stage of numerical computations the implicit scheme of the finite difference method is used, while the system of equations related to transition to the next time level solved by using the Gauss-Zeidl procedure. The formulas describing the temperature-dependent thermophysical paramers of metal (gold) are taken from [3]. In the final part of the paper the results of numerical computations and also the conclusions are presented.

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AN ISOGEOMETRIC FRICTIONLESS CONTACT FORMULATION OF COSSERAT RODS WITH UNCONSTRAINED DIRECTORS

Myung-Jin Choi[1], **Sven Klinkel**[1], **Sauer Roger A.**[2]

[1] Institute of structural analysis and dynamics, RWTH Aachen University, Germany
[2] Faculty of Civil and Environmental Engineering, Gdansk University of Technology, Poland

choi@lbb.rwth-aachen.de

We present an isogeometric finite element formulation of frictionless beam contact based on a Gauss point-to-surface contact algorithm with active-set iteration method combined with a penalty regularization. The beam formulation is based on unconstrained director kinematics of an arbitrary order, which enables an additive configuration update process, a straightforward utilization of three-dimensional constitutive laws without zero-stress conditions, and efficient description of cross-sectional strains. Further, a spatial discretization using NURBS basis functions enables a description of the lateral boundary surface with higher-order continuity, required to have continuous surface curvature and metric in the closest point projection. We verify the accuracy and efficiency of the proposed beam contact formulation by comparison with brick element solutions in numerical examples.

SOLUTION ALGORITHMS OF THE MULTIPOINT MESHLESS FDM FOR THE MULTISCALE AND NONLINEAR ANALYSIS

Irena Jaworska[1]

[1] Cracow University of Technology, Poland

irena.jaworska@pk.edu.pl

The novel higher order multipoint meshless finite difference method was developed recently. The method is based on arbitrary irregular cloud of nodes, the moving weighted least squares approximation, and the local or various global formulations of boundary value problems. The method provides p-type solution quality improvement and may be used to solve various types of engineering problems.

The paper presents the computational algorithms of the multipoint method application to such demanding engineering problems as numerical homogenization of heterogeneous materials and nonlinear, geometrically and physically, analysis.

Two basic versions of the multipoint meshless approach - the general and the specific ones were elaborated. The general formulation though it is more complex, may be used for all types of boundary value problems including nonlinear ones. The main advantage of this version is its generality - the basic relations of the derivatives from the unknown function are independent of a problem being solved and is based on a domain discretization only. This feature allows to introduce two stages of the multipoint computational strategy and is advantageous from the point of view of the calculation efficiency.

The appropriate algorithms of the method application to multiscale and nonlinear analysis, based on its characteristic features, were developed and examined, necessary procedures were written and tested. The paper is illustrated by several examples of the numerical analysis.

SOFTWARE ARCHITECTURAL ISSUES IN BUILDING IGA-BASED FLEXIBLE SIMULATION ENVIRONMENT IN C++

Anna Perduta[1], **Roman Putanowicz**[1]

[1] Chair for Computational Engineering, Cracow University of Technology, Poland

roman.putanowicz@pk.edu.pl

The purpose of the paper is the presentation of key ideas underlying the software architecture of an in-house package called AHIGA. The name stands for Adaptive Hybrid Isogeometric Analysis, and the main goal in designing and building it was to obtain a simulation platform for experimenting with various variants of IGA. IGA was introduced to bridge the gap between CAD geometric modelling and finite element technology through the utilization of NURBS as the base of fields approximation. From that perspective, it is possible to embed IGA in existing FEM software by tweaking the process of building shape functions and some tricks in the management of degrees of freedom and the assembly of a linear system. On the other end of the spectrum, building a single patch version of IGA software is also relatively simple. However, building a simulation engine that can fully exploit IGA potential, especially for general multi-patch geometries in the 3D within the context of adaptive simulations, warrants careful consideration of the underlying software architecture. The paper presents such considerations that resulted in software package featuring the following elements: single and multi-patch analysis in 2D and 3D, mesh refinement support in patches, independent selection of approximation spaces in each patch (Lagrange, Chebyshev, NURBS), patch 'glueing' and handling of boundary conditions via Nitsche's method, multi-field analysis with independent fields approximation, support for coupling different computational methods in different parts of the domain, support for non-linear analysis, specifically support for hyper-elastic materials. The paper describes the design of classes such as Model, MetaMesh, Discretisator, GeomMapper, BasisFunction, Variable, Brick, and Integrand, and their interplay in handling geometric models representation and discretization, switching approximation spaces, describing the known and unknown fields. Some issues in expressing the classes design in the C++ language are also indicated. The versatility and flexibility of the built simulation engine are illustrated on examples showing support for the following methods: GIFT (Geometry Independent Field approximation), DGFD (Discontinuous Galerkin Finite Difference), and FCM (Finite Cell Method). The potential of application of AHIGA package to practical engineering problems is illustrated by the example of solver for analysis of plates with multi-layer corrugated cores. Due to the use of NURBS for geometry modelling, it is easy to introduce parametric models and the advantages of the GIFT method are illustrated by comparing solutions obtained with the NURBS shape functions and Chebyshev ones.

COUPLING FEM WITH FD MESHLESS METHOD BY MEANS OF APPROXIMATION CONSTRAINTS

Jan Jaśkowiec[1], **Sławomir Milewski**[1]

[1] Faculty of Civil Engineering, Cracow University of Technology, Poland

jan.jaskowiec@pk.edu.pl

Over the past decades, two computational approaches of different type, namely the finite element method (FEM) and meshless methods (MM) were under rapid developments. Those two solution approaches have been found very effective in modeling variety of engineering. However, FEM and MM have their certain disadvantages that may be reduced or eliminated by appropriate coupling into one computational framework [1]. Among many possible versions of MM, the attention is laid upon the meshless finite difference method (MFDM).

The coupling of FEM and MM has a long-time tradition whereas many possible techniques have been proposed thought out the last two decades, including authors' original contribution to that issue [2]. In this research, we would like to present the new technique, which significantly differs from the ones reported in literature. The same technique may be applied to enforce the boundary conditions to the meshless approximation as well. The problem domain is divided into two disjoint subdomain, with the common boundary, to which FEM and MFDM frameworks are assigned. The approximation schemes in those two subdomains are not bonded in any manner, therefore the existence of discontinuities in the common boundary is the natural consequence. However, the final approximate solution has to be continuous over the entire domain. In order to solve this problem, degrees of freedom of both computational models may be combined in such a manner that the approximation function is continuous along the common boundary. In this paper, the relevant constraints subjected to degrees of freedom which guarantee such continuity are proposed. These constraints have the matrix-type form whereas the solution approach requires solving the strongly singular system of equations (SSE) first. The special technique is presented in this paper in order to find the complete solution of the SSE which allows to produce the constraints matrix. The same approach is applied to enforce the nonhomogeneous boundary conditions in the meshless subdomain.

The proposed approach is illustrated with several 2D and 3D examples, including the standard Poisson's problem as well as selected elasticity and thermo-elasticity problems.. In each case, it is shown that the new method of coupling FEM and MFDM is effective as it provides the accurate and continuous solution in the entire domain.

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A MIXED POLYGONAL FINITE ELEMENT FORMULATION FOR NEARLY-INCOMPRESSIBLE HYPERELASTIC MATERIALS

Bjorn Sauren[1], Sven Klinkel[1]

[1] Chair Structural Analysis and Dynamics, RWTH Aachen University, Germany

klinkel@lbb.rwth-aachen.de

We present a mixed extension to the displacement-based scaled-boundary formulation to overcome volumetric locking in polygonal elements with nearly-incompressible hyperelastic material laws. Both the displacement and pressure fields are parameterized by the scaled-boundary concept, in which the polygonal domain is splitted into a boundary and a scaling direction. In contrast to the semi-analytical scaled-boundary finite element method (SBFEM), the displacements and pressures in the scaling direction are herein approximated by Lagrangian interpolation functions. By doing so, arbitrary polygonal element geometries can be modeled and volumetric locking is alleviated. It is additionally shown that volumetric locking is no longer dependent on the location of the scaling center, which can be placed either inside or outside of the polygonal element. The star-convexity condition, which is mandatory for the SBFEM, is overcome by the present approach. Several numerical examples show that the method alleviates volumetric locking and the location of the scaling center can indeed be arbitrarily chosen. The obtained results for both regular and Voronoi meshes are in agreement with the analytical solutions and are compared to alternative polygonal element formulation, such as the mixed virtual element method (VEM).

ADAPTIVE LOCAL SURFACE REFINEMENT FOR ISOGEOMETRIC CONTACT, FRACTURE AND TOPOLOGY OPTIMIZATION

Roger Sauer[1]

[1] Gdansk University of Technology, Poland

roger.sauer@pg.edu.pl

Many applications in science and engineering are characterized by localized surface phenomena. These can appear in the geometry, such as surface wrinkles and cracks, in the constitution, such as material interfaces, and in the boundary conditions, such as contact. Further, the localized interfaces induced by these phenomena can move across the domain. Wrinkles and cracks can propagate, material interfaces can evolve, and contacting bodies can peel and slide. In the computational simulation of these examples it is advantageous to use adaptive local surface refinement. Such a refinement approach adapts the numerical discretization in both space and time to where and when it is needed most. In this they have to account for the underlying numerical description of the surface and its phenomena. The nature of many surface phenomena necessitates smooth discretizations such as those provided by isogeometric analysis.

This work presents an adaptive local finite element refinement strategy for surface phenomena based on NURBS discretizations, which provides at least C1-continuity between elements [1]. It uses a local representation of the parameter space based on the Bezier extraction operator [2] and the local refinement approach of [3]. Further, it allows for local coarsening of previously refined regions. The strategy is applied to the simulation of peeling and sliding contact, shell fracture, and surface topology optimization [1, 4]. The latter two cases are based on Kirchhoff-Love thin shell theory and high order phase field methods that both require C1-continuity for their efficient computational description. It is shown that the formulation is able to locally adapt the discretization and thus increase computational efficiency. In the case of contact, the formulation can be extended to 3D through the use of local surface enrichment [5]. It is also demonstrated that the proposed strategy can handle patch interfaces and domain discontinuities such as surface folds.

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