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S13 Shells and plates

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ANALYTICAL STUDY OF THERMO-MECHANICALLY LOADED FGM-STEEL/ALUMINUM HOLLOW SPHERICAL STRUCTURES

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Particularly in recent years the behavior of spherical containers and pressure vessels of functionally graded material (FGM) has been in the focus of numerous investigations. However, in each one thereof the basic approach is subject to specific presuppositions, of course. For example, in the majority of the studies the application of an appropriate homogenization procedure is avoided by assuming independent grading indexes for each material parameter. Moreover, many investigations take only either internal pressure or heating of the structure into account, and in several cases the results are based on a numerical approach. What is more, if specific material data of a real FGM are considered at all, the available studies give numerical results specifically for a metal/ceramic-FGM.

Nevertheless, nowadays also structures of metal/metal composites like steel/aluminum-FGMs are attracting more and more interest; in particular, they allow for significant weight reduction as compared to homogeneous steel devices while maintaining sufficient strength in many applications. Hence, it is the aim of the present contribution to give an in-depth discussion of spherical containers of steel/aluminum power-law graded material. The specific features of the investigation are the following ones. First, with regard to the load, both internal and outer pressure combined with elevated temperature is considered. Second, based on material data including the temperature dependence of the relevant properties of steel and aluminum (except for Poisson's ratio), it is shown that by appropriate functional approximations analytical solutions for stresses and displacements can be found for different homogenization schemes: remarkably, this holds true not only for Voigt but also for Reuss rule, the latter leading to hypergeometric functions. Third, stresses and elastic limits - based on von Mises yield criterion - are given for both full grading from steel at the inner surface to pure aluminum at the outer surface and partial grading from steel to a steel/aluminum alloy. Fourth, the achievable weight reduction in relation to the strength of the device under operating conditions is discussed thoroughly. Thus, an engineer is enabled to decide whether using a spherical steel/aluminum-FGM container may be advantageous for some applications; moreover, the results also may serve as benchmark solutions for purely numerical investigations.

INITIAL STABILITY ANALYSIS OF THIN PLATES IN TERMS OF THE FINITE DIFFERENCE METHOD

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The initial stability problem of thin (Kirchhoff-Love) isotropic and elastic plates supported on boundary is considered in the paper. The plate variable thickness with its linear variation is considered too. The Finite Difference Method (FDM) is applied to approximate the solution of the thin plate bending problem.
The corresponding derivatives of the plate stiffness appearing in the differential equation are calculated as derivatives of the complex function or alternatively expressed as appropriate differential quotients. The authors' algorithms for creating difference expressions were used [1]. It is also assumed that the complex external load acts on the plate in its plane, i.e. a plate is subjected to combined normal and tangential external loading. Various types of boundary conditions are considered too.
Several numerical examples were solved, and the obtained FDM results were compared with those obtained analytically [2] and numerically, e.g. by the Boundary Element Method (BEM) [3].

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GEOMETRIC FINITE ELEMENTS FOR DIRECTOR SHELL MODELS

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We consider the numerical discretization of geometrically exact director plate and shell models. Such models typically involve degrees of freedom that represent directions or orientations. These degrees of freedom do not form vector spaces, and are therefore notoriously difficult to approximate by finite difference or finite element techniques. In this talk we present geometric finite elements, a novel finite element method for problems with degrees of freedom in non-Euclidean spaces. Geometric finite elements introduce actual finite element function spaces, consisting of continuous functions of piecewise generalized polynomials. They have an elegant formulation, come with rigorous error theory, and fulfill all symmetry properties expected in mechanics. We explain the construction of geometric finite elements and demonstrate their properties numerically on different one- and three-director shell models.

VALIDATION OF THE CONTINUUM MODEL OF TENSEGRITY BEAM- AND PLATE-LIKE STRUCTURES

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In this paper, an attempt to derive a valid continuum model of tensegrity structures is made. The work consists of two parts. Firstly, the development of the continuum tensegrity model is provided and next, this model is validated. In contrast to the conventional truss systems, in the case of tensegrity structures, the proposed approach must include the influence of the self-stress. In this work, the effective properties of a single repeating unit (a tensegrity module) are obtained. These units are used to build tensegrity beam- or plate-like structures.

In an effort to develop the continuum model, the procedure called “the energy equivalency method” is adopted. The basis of that approach is the assumption that the finite element strain energy of a deformed tensegrity truss system contains the same energy as its continuum counterpart. In order to use the equality of energies, the nodal displacements of the discrete model need to be represented in terms of the strains of the continuum. A single repeating unit is isolated from the whole structure. The displacement field of that unit is assumed to be described by third-order polynomials and expanded in a Taylor series. The above procedure leads to the determination of the transformation matrix that enables to obtain effective stiffness properties of the continuum model.

After the derivation of the effective stiffness properties, the proposed continuum model is validated. This process completes the analysis and is very important for practical usage. The verification of the continuum model is done by the comparison of the displacements obtained for the continuum model and discrete one. The analysis of the continuum model is performed in the geometrically quasi-linear setting, while the consideration of the discrete models adopts a geometrically quasi- and non-linear approach. All numerical analyses are provided by using the calculation procedures written in the Mathematica environment.

The continuum model of tensegrity provides a simple tool for the analysis of large tensegrity structures, especially in case when many commercial software programs cannot be used. The finding of this work may also be useful for the analysis of metamaterials whose topology is based on the tensegrity concept.

NUMERICAL HOMOGENIZATION OF MULTILAYER WALLS OF TANKS FOR BIOMATERIALS

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Tanks are engineering facilities that have found wide application in many sectors of the economy in recent years. They are used not only to store loose materials, but also liquids. They include, among others tanks for clean water, bio-waste, tanks in sewage treatment plants or in the fuel industry. Liquid tanks are usually reinforced concrete tanks with an additional layer of thermal insulation and an external casing in the form of a trapezoidal sheet. The structures of such tanks are designed by analytical methods or by complex three-dimensional numerical analysis. However, in the strength calculations, only the monolithic supporting structure is taken into account. In addition, the creation of full 3D models of such structures is very time-consuming and requires a lot of effort and specialist knowledge, which is why simplified models are increasingly used in engineering practice. An example of such simplification may be the application of one of the homogenization techniques to the calculations. Numerical homogenization is an excellent tool that allows to simplify the calculation of the model and replace the complex structure with a homogeneous plate with equivalent parameters, which ensures that its behavior is very close to the three-dimensional reference model. The article presents a numerical homogenization procedure based on strain energy equivalence. Based on the proposed homogenization method, the structure of reinforced concrete tanks, consisting of a reinforced concrete load-bearing layer, a thermal insulation layer and a steel sheet outer jacket, is reduced to a plate element with effective stiffness. The above method allows for the correct homogenization of complex composite structures consisting of many elements with different material properties, without the need to use advanced numerical models. The only requirement is to build a full stiffness matrix of the representative volume element (RVE). Numerical calculations were performed for the models of tanks with and without taking into account the curvature of the tank, as well as for the axisymmetric case. In the homogenization process, not only information about different materials in the cross-section was used, but also their interconnection.

ON VIRTUAL ELEMENTS FOR KIRCHHOFF-LOVE PLATES AND SHELLS

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The virtual element method is based on an ansatz space in which the ansatz is only defined at the boundary. This feature permits to revisit the construction of Kirchhoff-Love (KL) plate elements of arbitrary shape. The C1-continuity condition is much easier to handle in the VEM framework than in the traditional finite element methodology.

We will show various VEM elements suitable for KL plates which are much simpler than the well-known TUBA finite elements. Based on a geometrically exact thin KL shell models, see [4], we will construct virtual elements for large deflections and compare these with a similar formulation for TUBA elements. The formulation contains new ideas and different approaches for the stabilization needed in a virtual element setting. In the case of C1-continuous elements it is crucial to use an efficient stabilization, otherwise the rank deficiency of the stiffness matrix associated with the projected part of the test function is more pronounced than for C0-continuous elements.
In this contribution we demonstrate how to construct simple and efficient virtual plate elements for isotropic and anisotropic materials.

The formulation will then be extended to geometrically exact shell elements. Various benchmark examples and convergence studies demonstrate the accuracy of the resulting VEM elements.

Finally, reduction of virtual plate elements to triangular and quadrilateral elements with 3 and 4 nodes, respectively, yields finite element like plate elements which are much simpler than TUBA elements. It will be shown that these C1-continuous elements can be easily incorporated in legacy codes and demonstrate an efficiency and accuracy that is much higher than provided by traditional finite elements for thin plates and shells.

DETERMINATION OF THE TORSIONAL STIFFNESS OF SANDWICH PANELS BY MEANS OF FULL-SCALE LABORATORY TESTS

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This paper presents the problem of behaviour of sandwich panels subjected to torsion. Typical panels used in civil engineering, consisting of two thin but rigid metal facings and a thick but shear deformable foam core, were analyzed. [1].

In common uses of sandwich panels, the torsion effect is rightly ignored. However, the negative effect of torsional forces may be significant in the case of an additional facade mounted to the sandwich panel building envelop or a supporting structure for various technological devices. In order to perform static calculations of sandwich panels subjected to torsion, it is necessary to know the torsional stiffness of the analyzed element [2].

In the laboratory tests [3], the deplanation of the twisted sandwich panels were partially blocked. This led to the disturbance of the free torsion by the effects of the warping torsion. The occurrence of such deviations increases the error in the determined torsional stiffness. In order to perform author's laboratory tests, an original test stand was designed to induce free torsion of sandwich panels. The sliding surface elements used in the construction of the research stand reduce the occurrence of local disturbances and facilitate the freedom of torsion of the tested samples. The torsional stiffness of sandwich panels were determined by measuring the relative increase of the angle of rotation of the cross-section of the twisted element along its length, in relation to the relative increase of the force generating the rotation. Sandwich panels with two different heights and three different widths of cross-section were subjected to torsion.

In order to evaluate the obtained experimental results, numerical analyses and analytical calculations were also performed. Numerical models were prepared in the SIMULIA Abaqus software. Three-dimensional models were prepared by using shells and solids finite elements. An isotropic model of the facing material was used, taking into account their layered structure. The core was modeled using an anisotropic material model. The material parameters were determined in laboratory material tests. In the analytical calculations, the one-dimensional beam model was used. The obtained results (experimental, numerical and analytical) were compared with each other, and the differences were identified and explained. The performed research allowed for a reliable determination of the torsional stiffness of the sandwich element.

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VIBRATIONS OF AN ORTHOTROPIC PLATE WITH POINT SUPPORTS SUBJECTED TO A MOVING FORCE

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We investigate the dynamic behavior of an rectangular orthotropic plate loaded with concentrated force P moving with constant speed v along the plate. Partial differential equation of motion describing damped vibrations of the plate is presented.

In this work we consider two types of plates in terms of boundary conditions. In the first case we assume that plate is simply supported on all of its edges and in the second one we take a look on the bridge plate with two simply supported and two free edges on the opposite sides of the plate. Solution for the first case can be presented in double sine series including eigenfunctions of the plate and functions determined from ordinary differential equations obtained by applying the Galerkin method. Solution for the second case can be obtained partially numerically by applying finite difference discretization of the plate along its length.

After obtaining solutions for the simply supported and bridge plate we can use them to describe vibrations of the plate with a number of arbitrarily located point supports. If we remove point supports and replace them with concentrated time-varying forces Xi(t) we will be able to describe vibrations of the plate by applying superposition rule.

Knowing that deflections of the plate at the coordinates of the intermediate point supports are equal zero we can build compatibility equations in order to determine forces Xi(t). For this purpose we shall apply Volterra integral equations for the case of simply supported plate and Newmark's method for the case of bridge plate.

Two numerical examples are given to prove effectiveness of the presented approach.

STRESS-BASED FINITE ELEMENT SOLUTION OF THE MINDLIN PLATE BENDING PROBLEM

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The bending problem for the Mindlin plate is solved by minimization of the functional of the complementary energy. To satisfy the equilibrium equations, the section moments are expressed by the Southwell stress function. As the shear forces are expressed as second derivatives of the stress function, the interpolation functions approximating the stress function are to be continuous together with their first derivatives to give the approximate solution converging to the exact solution. Two types of finite elements are applied: the Bogner-Fox-Schmit rectangular element and the Hsieh-Clough-Tocher triangular element. The equilibrium conditions on the domain boundary, along the elements edges and at its corners, have the form of linear constraints for degrees of freedom. These conditions are satisfied using the Lagrange multiplier method in a way shown in [1] in the case of the Kirchhoff plate model, where the isoparametric elements are applied.

Several examples of the static analysis are considered showing the convergence and high efficiency of the proposed approach. The results are compared with the results obtained by the displacement method in which the triangular, shear locking free element with parabolic and linear shape functions is employed to approximate the deflection and rotations, respectively. The upper and lower bounds for the strain energy are found by the present method and the displacement method, respectively. On the basis of two dual solutions: the stress-based and displacement-based ones, the error of the approximate solution is calculated using the Synge method [2].

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WRINKLING STRESS OF SANDWICH PANELS WITH NON-HOMOGENEOUS CORE

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In a typical sandwich element, the two metal facings are joined to each other by a relatively thick but deformable core. Due to its structure, the bending of sandwich panels corresponds to the action of a pair of forces acting on the facings. The facing that is in compression is prone to local loss of stability through wrinkling. It is the most common in practice mechanism of failure of sandwich panels.

In the literature, the problem of determining the value of stresses leading to the wrinkling of facing has been discussed for a long time. Classic approaches to the problem are based on the use of differential equilibrium equations [1] or energy methods [2]. Of course, numerous numerical and experimental approaches to the problem are also known. Predicting of the wrinkling stress value is a challenge when the facing and core materials are anisotropic [3] or heterogeneous [4].

This paper presents the results of experimental tests which show that the core of the sandwich panel is clearly heterogeneous. We know from the literature [5] that the properties of the core adjacent to the facing under compression have a decisive influence on the value of wrinkling stresses.

In this paper, two functions approximating the material parameters of the core are proposed, and then the expressions for wrinkling stress were derived using the energy approach. The derived equations were verified for selected parameters of the approximating functions. The obtained analytical results were also compared with the results obtained for the spatial numerical models.

The next stage of the work was to obtain a function describing the material properties of the core, which, for the given constraints, maximizes the value of the wrinkling stress. The paper presents the results that can be used in practice when developing new material solutions for sandwich panels industry.

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IMPLEMENTATION OF VISCO-HYPERELASTIC CONSTITUTIVE MODEL FOR SHELL ELEMENTS

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In this research, the constitutive law and its finite element implementation for shell elements is given. It is composed of an anisotropic, incompressible matrix with visco-hyperelastic formulation. The matrix is reinforced with fibers formulated using polynomial relation with viscous aspects. As the FEM platform the commercial, multi-purpose LS-DYNA code is used. The user subroutine UMAT is employed. It returns the components of Cauchy stress tensor that are fed into force resultants and couple resultants of the shell element formulation available in the code. The stored energy density function of the material is composed of two parts: elastic and viscous. The first one represents the stored elastic strain energy in the material, and the second part represents the dissipated energy obtained due to viscous aspects. For the description of the isotropic matrix, the Neo-Hookean strain energy function model with material parameters is employed. Fiber elongation is defined as a polynomial strain density energy function to prescribe fiber stretch. Additionally, the viscous part is governed by a polynomial function concerning the strain-rate dependent behavior. Simple linear regression is adopted to obtain the material mechanical constants of strain energy density function from the quasi-static and strain-dependent experimental data. The model is derived primarily to simulate large deformations of elements, and in this application dedicated to simulate deformations of human spine ligaments. To check the implementation of material model and its compatibility with the analytical solution, elementary simulation tests based on homogeneous prescribed motion are executed. Numerical simulations are conducted for simple tension, extension and shear tests. Additionally, various applications were presented, e.g. for different ligaments such as posterior longitudinal ligament, anterior longitudinal ligament, ligamentum flavum, capsular ligament, and interspinous ligament. Nevertheless, the model can be applied for other soft tissues that exhibit visco-hyperelastic responses. The material model considers and reveals the internal biomechanics of soft tissues of human body. In the conclusion we observe that this model can properly describe the visco-hyperelastic spine ligaments biomechanical behavior. Hence, the implementation of constitutive law can be successfully adopted for analyses that consider various ligamentous structures.

ANALYSIS OF FGM PLATES BASED ON NEUTRAL SURFACE AND GENERAL THIRD-ORDER PLATE THEORY

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One of the features of the FGM plates and problems to be solved is the position of the neutral surface which is shifted with regard to the mid-surface. A number of papers have treated this aspect. In the paper this problem was approached with the use of the general third-order plate theory (GTPT) developed by Reddy and Kim. Within the framework of the GTPT the equations of motion are derived and modified to uncouple in-plane and bending effects.
Validity of formulation is confirmed by a series of calculations of compressed plates resulting in buckling similar to structural response of isotropic plates and comparison with the results available in the literature.

NUMERICAL MODELING OF DELAMINATION IN MULTI-LAYERED PLATES WITH LARGE DEFORMATION DESCRIPTION

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Multi-layered structures are widely used in many branches of industry, for instance in automotive or civil engineering. In many cases they are exposed to severe conditions that can lead to their failure, mainly due to the decohesion between the layers. This process is called delamination. The numerical modeling of such a phenomenon is challenging [1]. An effective computer method for the non-linear analysis combined with a proper decohesion model must be used for reliable numerical modeling to trace the delamination process. In this work large deformation description is applied in a method that couples so-called FEM23 [2] and the discontinuous Galerkin (DG) method [3] for numerical modeling of the delamination in multi-layered plates. The FEM23 is a method that allows for a full 3D analysis of layered plates or shells using only 2D finite elements. The FEM23 uses spatial approximation which is a combination of the 1D transverse and the 2D in-plane approximation. In the standard version of the FEM23, the approximation in the transverse direction is continuous. However, in this paper the approximation is discontinuous. The discontinuities are located at the boundaries between the neighboring layers. In spite of the discontinuous approximation in the DG method, the displacements have to be continuous for the undamaged plate, which is enforced by the cohesion tractions acting between the layers. The physical model includes the reduction of the cohesion tractions when the relative displacements of the adjacent layers exceed a certain threshold, which leads to delamination. This approach, which combines the FEM23 and the discontinuous Galerkin method, is shown to be effective in the numerical modeling of the delamination process of multi-layered plates using the large deformation description. In the method, it is possible to analyze the situation when the delamination occurs at many places, showing their mutual interactions in the failure process. This method is illustrated with some examples where the multi-layered plates are subjected to external mechanical loads leading to delamination. These examples show how the failure proceeds when the delamination is initiated at many places of the plate.

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EVALUATION OF COMPOSITE AUXETIC ANNULAR PLATE UNDER LATERAL LOAD

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Presented problem considers the evaluation of the reactions of the composite, three-layered annular plate with auxetic properties of facings to the action of the lateral loads. The characteristic features of the auxetic material, which are invoked by the negative value of Poisson's ratio disclose the new, worth knowing responses of the composite structures. One of this structure can be the classic three-layered structure composed of thin facings and thicker soft core, where facings are loaded with normal stresses but the core with shear ones. Combination of the different materials of facings and core creates well known sandwich structure. The using of the auxetic material for outer layers of sandwich structure formulates the new problem, whose practical and scientific cognition is important. The annular plate is the examined object. Its auxetic outer layers are loaded with forces, which are regularly distributed on plate perimeter and act in the plane of the facings. The evaluation of the static responses of plate on the action of the radially compressed loads is the fundamental analysis. The paper [1] is the example in the literature of the work, where the problem of the evaluation of the stability behaviours of the annular sandwich plate with the auxetic core is undertaken. The auxetic effect has been examined taken into account various plate material, geometrical and connected with the support system parameters. The including of the various absolute values of the negative Poisson's ratio enables to conduct the repeated evaluation of the behaviours of plates [2], whose facings have new auxetic properties. The evaluation of the parameters of the critical state: loads and modes creates the image of the buckling behaviours of auxetic-foam-auxetic composite plate. The comparison of responses of auxetic plate and plate with conventional elastic facings is the additional observation. The calculations have been carried out for two plate models: analytical and numerical with the usage of the approximation methods: orthogonalization and finite difference and numerical one built of the finite elements. The character of the responses of both plate models is consistent. The increase of the critical loads of auxetic plates with the increase of the absolute value of Poisson's ratio is the one of the result observation.

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ON THE MATHEMATICAL MODEL OF CIRCULAR/ANNULAR PLATES RESTING ON WINKLER FOUNDATION

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Circular/annular plates resting on elastic foundations are extensively utilized in engineering applications. Practical examples involve footings along with raft foundations of various structures such as liquid storage tanks, process tower bases, base of silos, and chimneys, to mention a few. Winkler put forward a simple model to idealize the behavior of elastic foundations supporting beams and plates. In the original Winkler model, the modulus of subgrade reaction, denoted by k, is assumed to be uniformly distributed beneath the superstructure. However, there are examples where the assumption of a uniform modulus is unrealistic, and a more robust analysis requires considering the inhomogeneity of the subgrade. With due attention to practical importance and theoretical significance of the problem, the current study provides an analytical procedure to determine the dynamic response of a thin circular/annular plate resting on a spatially inhomogeneous foundation. Singular points are made amenable by introducing the Frobenius power series, thereby permitting the use of more-general functions to describe the variation of the foundation modulus. Moreover, the proposed method is proved to be applicable for any type of classical boundary condition (simply-supported, free, fixed, elastically-supported, etc.). A parametric study is then conducted in order to investigate the variation of the frequency parameter for a general linear variation of the subgrade modulus. In conclusion, the application of the proposed method to circular/annular plates resting on a two-parameter (Pasternak) foundation with an inhomogeneous spring layer is set forth.

Keywords: Spatially-inhomogeneous Winkler foundation, Circular plate vibration, Singular points, Frequency parameter, Variable two-parameter foundation.

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