

## S16 Structural optimization and optimum material design

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## THE ISOTROPIC MATERIAL DESIGN OF THE ELASTO-PLASTIC STRUCTURES

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The Isotropic Material Design method (IMD) solves the problem of optimum distribution of elastic moduli of isotropy to make a structure as stiff as possible, see Czarnecki (2015) for the single load variant and see Czarnecki and Lewiński (2017) and Lewiński (2021) for the multiple load cases. Due to bulk and shear moduli being independent design variables, the essential part of the optimal structure turns out to exhibit auxetic properties, as expected by Lim (2015). The paper puts forward a new version of the IMD method for optimum design of structures made of an elasto-plastic material within the Hencky-Nadai-Ilyushin theory. The trace of the Hooke tensor represents the unit cost of the design. The yield condition is assumed as independent of the design variables, to make the design process as simple as possible. By eliminating the design variables the optimum design problem is reduced to the pair of the two mutually dual Linear Constrained Problems (LCP), see Bołbotowski (2021). The solution to the LCP stress-based problem determines directly the layout of the optimal moduli. A numerical method has been developed to construct approximate solutions, which paves the way for constructing the final layouts of the elastic moduli. Selected illustrative 2D solutions are to be reported, corresponding to various data concerning the yield limit and the cost of the design, see Czarnecki and Lewiński (2021). The yield condition introduced in the paper results in bounding the values of the optimal moduli in the places of possible stress concentration, like reentrant corners.

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## GEOMETRY OPTIMIZATION OF TRUSSES BY FORCE DENSITY METHOD

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This work concerns geometry optimization of truss structures. More specifically, our goal is to minimize the volume of a truss by adjusting the position of nodes (member joints) within a given design domain while assuming that: i) topology of the truss remains unchanged in the optimization process, ii) member forces are in static equilibrium with the external load, and iii) member stresses achieve the limit value in tension or compression.

Employing the idea of force density, that is a ratio of member force to member length, allows for reformulating the objective function and constraints as linear and quadratic polynomials in five indeterminates: three vectors of nodal coordinates and two vectors of force densities. Detailed explanation of the latter is dropped here, we only mention that introducing two (instead of one) force density vectors helps circumventing certain computational issues associated with volume minimization of fully stressed trusses.

Thus formulated, finite-dimensional variational problem is nonlinear and non-convex in general. Such shortcoming is usually associated with geometry optimization methods, and absent in algorithms using linear programming for topology optimization, where the location of nodes is fixed. This, in turn, means that numerical strategies for geometry optimization carry large computational cost, especially for trusses that are heavily populated with members and nodes. Consequently, our approach is not a practical choice in the numerical study aimed at predicting the layout of Michell structures.

On the other hand, however, limiting the scope of computations to structures with a relatively small number of design variables compensates (to some extent) for long CPU time. This makes the numerical strategy proposed in this study suitable for rational solutions, which can be easily transformed into manufacturable projects with clearly arranged members.

Geometry optimization methods are also useful in tackling the optimal form-finding problem, that is a design challenge of finding such a structural form, which optimally adjusts to the profile of the applied load. This goal is naturally achieved by relaxing the constraints on spatial location of loaded nodes, e.g. by allowing them to be positioned anywhere in a prescribed region instead of fixing them at given points.

The conference paper contains short theoretical introduction to the problem and discusses several optimal solutions in 2- and 3-dimensional design spaces.

## RECOVERY OF MICROSTRUCTURES APPEARING IN THE LEAST COMPLIANT 2D NON-HOMOGENEOUS ELASTIC BODIES

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The two new methods of the recovery of the microstructures for given nonhomogeneous optimal layouts of bulk and shear moduli (equivalently Young modulus and Poisson's ratio) maximizing the stiffness of a 2D isotropic elastic body are proposed. The microstructures are recovered by matching the optimal elastic moduli values predicted by the stress-based isotropic material design method (IMD) with the values of the effective moduli of the representative volume elements (RVE) calculated by asymptotic homogenization of periodic media. The microstructure topology is described by parametric description of single or several fibers in RVE or by pin-jointed (i.e. bending-free) lattices with a nonuniform topology. The used hexagonal shape of the RVE with its specific internal symmetry provides the assumed isotropy of the periodic body. The periodicity of the structure and the final optimal topology of the material is ensured by multiplication of a single fiber in accordance with the symmetries adopted for RVE or by combining the 3 or 12 pin-jointed families of pin-jointed bars with different stiffness characteristics into two or three groups with identical bar stiffnesses. The presented study is limited to the two-dimensional fibers (plates) with freely parameterized shapes of the curvilinear boundaries inside the RVE or to the truss lattice microstructures (bending-free pin-jointed lattices) with different longitudinal stiffness characteristics. In general, a truss model ignores the influence of the physical dimensions of the nodes and the bending effects which appear for lattice structures, but in both cases, the proposed methods allow for the exact reproduction of optimal Young modulus and Poisson's ratio found for each point of the body by IMD method, i.e. allow to construct properly graded optimal microstructures. A very important feature of both methods is the ease of generating microstructures with the desired elastic properties at any point in the design domain, which change in any directions in its neighborhood in accordance with the values of the components of the heterogeneous isotropic Hooke tensor found by the IMD method. The ability to model the auxetic, heterogeneous isotropic structure (meta-material with negative Poisson's ratio) is also shown.

# APPLICATION OF RADIAL BASIS FUNCTIONS IN LEVEL-SET METHOD FOR STRUCTURAL TOPOLOGY OPTIMIZATION

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In this paper we implement numerical method for shape and topology optimization of two-dimensional linear elastic structures. The method is based on the concept of shape derivative and the level-set method using radial basis functions and applied to compliance minimization problem with weight constraint. The cost functional depends on shape through the displacement state which is solution of linear elasticity system. In numerical approach we set design domain in which all admissible shapes are included.

Level-set method provides the feasibility of capturing the shape of structure on fixed mesh as a zero-level set of implicit higher-dimensional function. The evolution of level-set function (LSF) is governed by Hamilton-Jacobi equation, so the shape evolves in time with given normal velocity field computed in term of shape derivative. The conventional approach requires dealing with Partial Differential Equation (PDE) using for example explicit first order upwind differencing scheme on Cartesian grid. To keep the LSF sufficiently regular, it is also recommended to perform so-called reinitialization procedure from time-to-time, which consists in solving a properly modified PDE.

Set of real-valued axial symmetric functions known as Radial Basis Functions (RBFs) are commonly used to approximate a given function. Following Wang and Wang (Int. J. Meth. Engng 2006, 65:2060-2090) we use globally supported multiquadric splines to construct an implicit LSF. This function is a linear combination of RBFs centered at  $N$  knots and controlled by properly calculated coefficients. Evolution equation (PDE) is transformed into a system of ordinary differential equations (ODEs) thanks to substituting the RBF-LSF into original Hamilton-Jacobi equation and using collocation method. Shape of a structure is now updated through the coefficients obtained with solving the system of ODEs in each iteration.

The program is implemented in Wolfram Mathematica. To improve the efficiency we enforce approximate normalization procedure near the boundary and apply a smooth function which bounds the values of LSF. These actions ensure appropriate properties of LSF and enable better numerical stability. We employ so-called ersatz material approach and density-based mapping of the geometry to a mechanical model. Making use of the same mapping we also figure body forces in optimization process. For comparison purpose we have implemented 4-node and 8-node plane stress finite elements. We also study the influence of collocation points positions on the optimal solution. According to the performed benchmark tests, we can conclude that the developed program is very stable and not too much sensitive to mesh density.

# THE NOVEL EFFICIENT HEURISTIC STRUCTURAL TOPOLOGY GENERATORS FOR ENGINEERING APPLICATIONS

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Over years, the topology optimization has been a dynamically developing research area with numerous applications. This progress has been stimulated by the implementation of innovative, efficient and versatile optimization approaches and methods. Simultaneously, the practical aspect of engineering implementation of topology optimization techniques has become more important. As the result, the topology optimization tools are nowadays present in commercial engineering software like Ansys, Altair-OptiStruct, NX-Nastran or Abaqus-Atom. However, the black-box topology generators implemented into commercial software do not guarantee that the final results are the best available. Therefore, although remarkable achievements have been already made towards topology optimization application in engineering, there is still space for further investigations. It seems that the combination of professional FE structural analysis code with a newly developed efficient optimization software can give in many cases better results. In the present study the newly developed heuristic algorithms to generate optimal structural topologies are introduced.

The hybrid algorithm is discussed first. The main idea that stands behind this proposal is to take two existing algorithms and to build a new one which takes the advantage of both algorithms capabilities, what finally allows to improve efficiency of optimal topologies generation process. As to the algorithms choice, the first one is based on the formal optimality criterion whereas the second one utilizes a special heuristic rule of design variables updating. In numerical implementation of the hybrid algorithm, the design variables are updated at each iteration using both approaches and the better one from obtained solutions, in terms of objective function value, is selected for the next iteration. The proposed hybrid technique based on switching between the considered rules allows to obtain final structures having lower values of compliance as compared with results of application of basic algorithms running separately.

Another proposal it is the original concept of cellular automaton. The main idea that stands behind this proposal is to take the advantage of colliding bodies phenomenon and use the governing laws to derive original cellular automata rules which can efficiently perform optimal topologies generation process. It is worth underlining that the rules are built so as to cope also with irregular finite element meshes.

The proposed algorithms have been combined with commercial finite element software package Ansys to illustrate their versatility. Selected 2D and realistic 3D engineering problems have been investigated within numerical tests of generation of minimal compliance structures. Based on promising results of the tests performed, one can consider the proposed concepts of the algorithms as the alternatives for other existing topology generators for engineering applications.

## MODELLING OF AUXETIC PHASE COMPOSITE MATERIALS WITH OPTIMIZED PROPERTIES BY FAST MULTIPOLE BEM

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A novel group of materials are composites with auxetic phases that have a negative Poisson's ratio. Combinations of auxetic phases with positive Poisson's ratio phases have been found to provide optimal effective elastic properties. In the literature, topology optimization results are available, for example, optimizing Young's modulus in terms of the volume fraction and shape of the constituents. As a result, composites with shapes close to spheres or cubes are found, depending on the volume fraction. Such materials are typically modelled by the finite element method (FEM), which requires discretization of the whole domain of the representative volume element (RVE).

In the present work, homogenization problems involving auxetic phase composites are considered. RVEs are analyzed using the fast multipole boundary element method (FMBEM) for 3D composites with linear elastic constituents. The main advantage of the method is the discretization of the boundary of the analysed structure instead of the whole domain. In contrast to conventional BEM, the FMBEM offers linear (or quasi-linear) computational complexity. The developed method employs higher-order elements and adaptive integration to achieve high accuracy. The BEM many-inclusion formulation is applied, which employs the assumption of displacement continuity and traction force equilibrium at the interfaces. Boundary elements with quadratic shape functions are applied. Recent results show that the method can efficiently analyse a wide range of constituent elastic constants and the reinforcement volume fraction. In the FMBEM, the discretization of the boundary allows one to easily model different shapes and reduces the number of degrees of freedom. In the present work, the FMBEM is applied to the modelling of 3D composite materials with auxetic constituents. The results obtained by the FMBEM agree with the literature. The FMBEM can be applied to the optimization of such materials.

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## TOPOLOGICAL SENSITIVITY OF HIGHER ORDER HOMOGENIZED TENSORS IN PERIODIC ELASTICITY AND APPLICATIONS

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In this lecture we study and apply topological optimization techniques in order to synthesize new architected materials in the framework of homogenized continuous materials in 2D. The shape and topology optimization of microstructures can be performed using the topological derivative method.

We recall the context of homogenized linear elasticity for periodic materials in 2D, resulting in 'higher order media'. We interest in particular in 'second gradient media'. Higher order homogenized elasticity tensors are formally defined, from the two-scale asymptotic expansion method and the expression of the strain energy average on the unit cell.

Then, the sensitivity analysis of these higher order homogenized elasticity tensors to topological microstructural changes is performed. The microstructure is topologically perturbed by the nucleation of a small circular inclusion of weak material that allows for deriving the sensitivity in closed form, with the help of appropriate adjoint states, in the framework of periodic Sobolev spaces. The resulting topological derivatives measure how the higher order homogenized elasticity tensors change when a small circular inclusion is introduced at the microscopic level. The expression of the topological derivatives are explicitly given, with the use of corresponding adjoint states, and of polarization tensors.

Finally, the obtained topological derivatives of higher order homogenized elasticity tensors are used within a numerical method of shape and topology optimization of microstructures. The topological derivatives of functionals for multiscale models can be obtained and used in numerical methods of shape and topology optimization of microstructures, including synthesis and optimal design of metamaterials by taking into account the higher order mechanical effects. Thus, we obtain new topologies at the scale of the periodic cell that allow for example the design of periodic materials with pronounced second gradient properties.



## MODELING THE MECHANICAL PROPERTIES OF MICROALLOYED STEELS CONTAINING Nb AND V ELEMENTS

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Microalloyed steels with ferrite-pearlite microstructure are important group of structural steels, where high mechanical properties are achieved through small additions of elements such as Ti, Nb, V introduced separately or comprehensively. High chemical affinity of these elements for interstitials (N, C) results in precipitation of binary compound: nitrides and carbides and products of their mutual solubility - carbonitrides. In steel containing one of the microalloying elements (Ti, Nb, V) the carbonitride described by chemical formula  $MCyN_{1-y}$  is formed, where  $y$  means atomic fraction of carbon in carbonitride. These compounds have a dual role. The carbonitrides undissolved at austenitisation temperature inhibit the growth of austenite grains, providing a fine grain of supercooled austenite transformation products. The second most important factor influencing the mechanical properties of microalloyed steel is the effect of strengthening of ferrite by dispersed carbonitrides precipitations formed during the transformation austenite-ferrite as a result of reactions between elements dissolved in austenite.

Knowledge of parameters carbonitrides precipitations, both undissolved in austenite at high temperatures and formed in ferrite during phase transformations of undercooled austenite allows to predict the mechanical properties after manufacturing process using the knowledge of the steel chemical composition and process parameters. Carbonitride precipitations parameters, their contents, volume fraction and size distribution of precipitates can be calculated using mathematical models.

The aim of the work was to implement and use the developed kinetic model of the carbonitrides precipitations process in microalloyed steels to show the influence of microalloying elements V, Nb and interstitials N, C on the stereological parameters of carbonitrides precipitations in microalloyed steels and their mechanical properties.

The methodology of use the own computer program DEFFEM3D and possibility of modern thermomechanical simulators Gleeble 3800 to show modeling of the high temperature mechanical properties of steels was presented.

## OPTIMIZATION STRATEGY FOR THE INVERSE ANALYSIS OF MATERIAL STOCHASTIC MODEL IDENTIFICATION

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Development of various industries is associated with the search for new construction materials combining high strength with good formability and a high strength-to-density ratio. These features can be obtained for modern steels with heterogeneous microstructures, where hard constituents are dispersed in a soft and ductile matrix [1]. Advanced numerical models are needed to gain knowledge on distributions of microstructural features and resulting properties. The material model, which includes internal stochastic variables in the evolution equation was proposed in [2]. Dislocation density and grain size were defined as stochastic variables and distributions of these variables were obtained instead of average values, what is crucial for description of heterogeneous microstructures.

Reliable simulations of material processing require quantitative evaluation of model coefficients for considered material. The coefficients estimation for classical models, called an inverse analysis, is well-known [3]. Since the problem is ill-posed, the inverse solution is transformed to optimisation task with an appropriate objective function. The formulation of that function is a vivid problem when the model output is a histogram. A method dedicated to inverse problems for stochastic models was proposed in [2]. Since solutions obtained for individual Monte Carlo points may be entirely different, the selection of the numerical parameters of both the model and the optimization method is important. Thus, numerical tests, which aimed at evaluation of the influence of the numerical parameters on the model's output and its performance, are the objective of the work. The computation time is an impactful factor in terms of optimisation techniques used during the inverse analysis, which require several thousands of model recalculations to achieve a satisfactory result. Therefore, the preliminary step was to find the optimal number of solutions and bins. Various measures of the distance between histograms in the objective function were analysed and Bhattacharyya measure [4] was selected. Various optimization methods were tested and the best optimization strategy was proposed.

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# PHASE FIELD TOPOLOGY OPTIMIZATION OF ELASTO-PLASTICITY WITH FRICTIONAL CONTACT

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Contact problems appear in many different fields of mechanical engineering. The modeling and the optimization of these nonlinear problems is an important topic which is investigated in many laboratories. Most research related to shape or topology optimization has been concerned mainly with the linear elastic structures. Only a small part of the published papers has been concerned with the structural optimization of elastic - plastic structures. In many industrial applications the ability to take into account nonlinear mechanical behaviours of the structure is of great interest. In contacting surfaces the abrupt change in the mechanical or thermal properties of the materials or friction may result in stress concentration, degraded bonding strength or plastic deformations.

This work aims to analyze and to solve numerically the topology optimization problem for two bodies in bilateral frictional contact. The small strain elasto-plastic material model with linear kinematic hardening rather than pure elastic material model is assumed. The contact phenomenon is governed by the system of the coupled variational inequalities in terms of the displacement and the generalized stress. The structural optimization problem consists in finding such material distribution of the body in contact to minimize the maximal contact stress and to ensure the uniform distribution of this stress. The material distribution is described by the phase field function governing the concentration of solid and void sub-domains. Ginzburg-Landau functional is added to the original cost functional to ensure the boundedness of the design domain perimeter.

The original system of coupled variational inequalities is transformed into the system of nonlinear equations using the regularization and penalization techniques. The Lagrange multiplier technique is used to formulate the necessary optimality condition. The gradient flow equation is formulated to describe the evolution of the domain occupied by the body in the design space. This equation is numerically solved to find the optimal topology. Generalized Newton method combined with the phase field method are used to solve numerically this optimization problem. The results of computation are provided and discussed. The elasto-plastic optimization results are compared to pure elastic design results. The comparison shows that structures designed with accounting for plastic deformation have a reinforced area where plastic deformation occurs.

# OPTIMAL DESIGN OF PLANE ELASTIC MEMBRANES AND APPLICATION TO 3D FORM-FINDING

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Recently, in the paper [1], a novel problem of optimal design of plane membranes was put forth. Therein, one varies the membrane's pre-stress tensor field, which puts the formulation in the class of plastic design methods. The present talk virtually brings this idea to elastic membranes modelled by Föppl's theory [4], where the in-plane strain is expressed via the operator known from von Kármán's plate theory: the strain is linear with respect to the in-plane deformation and non-linear with respect to the membrane's deflection. Our goal is to find a distribution of the membrane's thickness that - under the constraint on the volume - minimizes the compliance for the out-of-plane load. This convex design problem proves to be well-posed, and it gives rise to a pair of mutually dual convex variational problems of the form that was studied in [1]: the stressed based problem (P) and the displacement based problem (P\*). From the mathematical perspective this pair is reminiscent of the one known from the Michell theory, or from the Free Material Design problem [3]. The proposed design setting is general enough to consider a whole class of non-linear constitutive laws that rules out compression of the membrane. The numerical strategy for the optimal membrane problem consists in discretizing the pair (P), (P\*). This is achieved via the finite element method that is newly developed and tailored specifically for this pair.

The paper [2] showed that the optimal design of 2D plastic membranes paves a way to optimal 3D form-finding, which allowed to efficiently identify precise plastic grid-like approximations of optimal vaults. In this talk we follow this approach for the optimal elastic plane membranes. We prove that the surface on which the optimal form should concentrate coincides with the deformed optimal membrane, and one recovers the material distribution through unprojecting the membrane's thickness. Effectively, the foregoing finite element method furnishes suboptimal shells that approximate the optimal 3D form.

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## SELF-ADAPTIVE POPULATION RAO ALGORITHM FOR OPTIMIZATION STEEL GRILLAGE STRUCTURES

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In this research, the self-adaptive population Rao algorithm (SAP-Rao) is used to obtain the optimum design of steel grillage structures. By using cross-sectional area of W-shapes as design variables, the constrained size optimization of this type of structure was carried out with the LRFD-AISC. The objective function of the handled problem is to find minimum weight of the grillage structure. The maximum stress ratio and the maximum displacement in the inner point of steel grillage structure are taken as the constraints. To calculate the moment and shear force of the each member and calculate the joint displacement, the finite elements method (FEM) was used. The developed computer program for the analysis and design of grillage structure and the optimization algorithm for SAP-Rao are coded in MATLAB. The results obtained from this study are compared with the previous works related to grillage structures. The results show that the SAP-Rao algorithm presented in this study can be effectively used in the optimal design of grillage structures.

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## MULTI-OBJECTIVE OPTIMIZATION OF PATIENT-SPECIFIC ARTIFICIAL HEART VALVES

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Aortic valve dysfunction is one of the most frequently detected diseases in the cardiac system. Artificial heart valves play a key role in the treatment of patients with these diseases. The development of artificial heart valves has been tremendous over the years, but there are still many challenges and problems to be solved, such as improving the durability of the structure and its dimensional adaptation to the patient [1].

The study aimed to develop new solutions for the design of aortic heart valves with the use of simulation and optimization methods [2-3]. In the study, a numerical model of the aortic valve was created, divided into the most important areas: free margin region, belly region, attachment region. For individual areas, an attempt was made to determine the sizing parameters as well as gradient-functional material parameters of composite material.

For optimization studies, the parametric model was combined with the structure simulation environment, where the multi-stage pressure load was applied. Moreover, the concurrent parametric-numerical model was used for multi-objective optimization using genetic algorithms. To determine the optimal geometric parameters and the structure of the gradient-functional material two objective functions, the minimization of the structure volume as well as the minimization of stress were accepted. In addition, a set of design variables describing the number, thickness, and directionality of the layers of the composite material was accepted.

Optimization tests were performed for three different models of the structure, assumingly adapted to different patients. As a result of optimization studies, three optimal designs were obtained, adjusted to the patients.

The design methodology presented in the article allows for designing aortic valves based on gradient-functional structure gives a chance to create a prosthesis that will be resistant to degradation processes by reducing areas with increased stresses.

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## ADVANCED EVOLUTIONARY PROCEDURES OF DESIGN FOR ADDITIVE MANUFACTURING

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The use of topological optimization to design additive manufactured structures is very popular in the industry today. However, despite the development of CAD / CAE tools, there are still many problems, including the application of technological constraints and the need to meet other advanced design requirements (e.g. specific thickness of structural components).

The article presents a set of new procedures of evolutionary topological optimization dedicated to the design of additively manufactured structures. The evolutionary topological optimization algorithm has been enriched with procedures for controlling the cross-sectional area of structural elements, which, acting as a limitation of the construction space, affect the form of the obtained structural solutions. The qualitatively new spatial solutions created in this way are highly suitable for additive manufacturing.

The new design procedures are illustrated with test examples that take into account the stress limitations. As the tests carried out showed, the solutions obtained with the new method were characterized by low weight and high stiffness.

## STRUCTURAL TOPOLOGY OPTIMIZATION OF A MODULAR SNAKE-LIKE MANIPULATOR

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This contribution concerns a snake-like robotic manipulator arm proposed first in [1]. The manipulator is composed of linearly but nonaxially joined identical modules with a possibility of relative twist, which amounts to one degree of freedom per module. It is an example of an extremely modular system [2], and its advantages are: economization (due to modularity and possible mass production) and robustness (easy repair by replacement of a failed module). The hitherto research involved the possible geometric transformations of the manipulator arm [3], but not its structural optimization. However, structural design of the involved modules is a challenging task, as the process has to take into account the relative position of the module along the arm, as well as the variety of global configurations of the deployed manipulator. It leads to a multi-load structural optimization problem with a significantly large number of loads. This contribution considers topology optimization of such a modular manipulator structure. Due to the large variety of possible load conditions, the initial analysis involves a 2D model of the structure with a discrete set of two possible relative arrangements of adjacent modules. Such a formulation allows the proposed approach to be preliminarily explored, tested and optimized in a numerically manageable simplified environment.

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# THE NEW PARADIGM FOR LIGHTWEIGHT DESIGN - BIOMIMETIC APPROACH TO COMPLIANCE MINIMIZATION

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In the paper the new paradigm for structural optimization without volume constraint is presented. Since the problem of stiffest design (compliance minimization) has no solution without additional assumptions, usually the volume of the material in the design domain is limited. The biomimetic approach, based on trabecular bone remodeling phenomenon is used to eliminate the volume constraint from the topology optimization procedure. Instead of the volume constraint, the Lagrange multiplier is assumed to have a constant value during the whole optimization procedure [1]. In case of the SIMP method (Solid Isotropic Material with Penalization) the optimization task is carried out using the finite element method, where the parameter vector is identified with the density distribution of a fictitious material assigned to each of the finite elements. This density determines the stiffness expressed by the Young's modulus values derived with use of the heuristic procedure for upgrading the density distribution. While effective, this approach has some drawbacks like identical topologies resulting from the optimization procedure for different load magnitudes. Presented biomimetic approach does not have these drawbacks, but it also requires the development of appropriate heuristics, which will be presented in the paper. With the use of the new optimization paradigm, it is possible to minimize the compliance for multiple loadcases [2] by obtaining different topologies for different materials. It is also possible to obtain different topologies for different load magnitudes [3]. Both features of the presented approach are crucial for the design of lightweight structures, allowing the actual weight of the structure to be minimized. The final volume is not assumed at the beginning of the optimization process (no material volume constraint), but depends on the material's properties and the forces acting upon the structure.

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# AN EFFICIENT METHOD FOR TOPOLOGY OPTIMIZATION WITH PRESCRIBED SAFETY MARGIN

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To solve the problem of reliability-based topological optimization, a heuristic algorithm is used, consisting in removing redundant material in the areas with the lowest stress intensity. In this algorithm, the design variables represent the material densities in the individual finite elements. The material is removed by reducing the density of finite elements as a function of the stress intensity.

A side effect of the optimization process is a reduction in the safety of the structure. This is especially important in topological optimization where the process requires material removal [1]. Providing a safety margin consists in introducing additional constraints related to the probability of failure to the formulation of the optimization problem. This means that the probability of failure will not be greater than the explicitly indicated value.

However, adding a reliability analysis to the optimization process can significantly increase its computational effort and therefore, the use of efficient methods to determine the failure probability becomes a key issue [2]. The low probability of failure, which should characterize a safely designed structure, can be estimated with sufficient accuracy by applying first-order methods. This approach is based on the assumption that the objective function can be linearly approximated in the vicinity of the design point. A large number of algorithms based on first-order assumption has been developed for the evaluation of reliability constraints, among which Reliability Index Approach (RIA) and Performance Measure Approach (PMA) are most frequently used. The latter approach is characterized by better numerical efficiency and stability.

The effectiveness of the proposed methodology will be illustrated with several numerical examples. The originally developed computer program written in MATLAB environment [3] will also be briefly presented.

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## OPTIMAL DESIGN OF ARCHGRIDS: FINITE ELEMENT APPROACH

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In this work new numerical approach for constructing of optimal archgrids is proposed. The archgrids, discovered by William Prager and George Rozvany in 1970s, are viewed here as tension-free and bending-free, uniformly stressed grid-shells forming vaults evenly supported along the closed contour of the basis domain. The optimal archgrids are characterized by the least volume. The optimization problem of volume minimization is reduced to the pair of two auxiliary mutually dual problems. In papers by Lewiński, Dzierżanowski and Czubacki trial functions for both primal and dual variational problems are decomposed in two function bases: trigonometric (Fourier) and polynomial (Legendre). The focus was on structures composed of arches forming a rectangular grid, i.e. running in two mutually perpendicular directions and spanning a given rectangular domain. In the paper by Dzierżanowski and Hetmański the general variational framework of the optimization problem of optimal archgrids was reformulated in the discrete setting and the computations were based on the second-order cone programming (SOCP) implemented in Matlab. In that approach the design variables are calculated at the points that lie on the arches.

In a case of the archgrids made of arches oriented by 0, -45, 45, 90 degrees with respect to some reference axis the design domain can be discretize by the orthogonal grid. By each point of the grid the arch representing each direction go through without increasing the number of unknowns. If we add new family of arches with some arbitrary direction, to satisfy this condition, the number of unknowns will increase rapidly.

In new approach proposed in this work the design domain is divided into triangle finite elements with linear shape functions. The unknown design variables are located in the nodes. The mesh is independent from the families of arches which are used only for calculating the optimization constrains called mean square slope condition. For each arch its elevation function is calculated in the points being the intersections of the arch with the edge of the finite elements. To obtain proper solution each unknown has to be present in the constrains. It means that through each finite element at least one arch has to go through. The new approach was implemented in Mathematica software and was used to calculate new examples of archgrids with families of arches going simultaneously in many directions and being composed over non-convex design domain e.g. cross domain.