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Overall microstructure response function and its application to recovery of microstructure

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The aim of the homogenization is up-scaling of mathematical description of the process under consideration, from the scale of heterogeneities to the scale of engineering applications. For the linear problems, the micro- and the macro- descriptions are analogous in the mathematical form, except material properties involved in both descriptions. The material properties of the micro-description are space dependent whereas that of the macro-description, called as overall ones, are constant since they characterize macroscopically homogeneous medium.

Two mathematical problems are called inverse to each other if the formulation of the first problem contains the solution of the second problem and vice versa. The evaluation of overall properties in terms of the phase properties and the microstructure morphology can therefore be interpreted as the direct problem of homogenization since it consists of projections from the known microstructure morphology. The recovery of the microstructure morphology using values of the overall material constants is therefore the inverse problem of homogenization.

The inverse problem, in general, has no unique solution. In order to ensure the existence and uniqueness of the solution the problem has to be supplemented by a definite set of appropriately chosen overall material constants values as well as one has to postulate the particulate type of microstructure morphology. The set of overall material constants values may also be considered as a projection of material constants of composite components onto the space of admissible values of overall material constant; it can be also interpreted as a macroscopic manifestation of the medium microstructure and is hereinafter referred to as the overall microstructure response function.

Two type of microstructure morphology are postulated to solve the inverse problem, i.e. randomly oriented spheroidal inhomogeneities of certain distribution of the aspect ratios embedded in the matrix and a binary mixture. In this context the recovered microstructure has to be interpreted as an equivalent or replacement one since it does reproduce the overall microstructure response function but not necessary the original microstructure morphology.

For the microstructure morphology of randomly oriented spheroidal inhomogeneities, the inverse problem is formulated as a linear Fredholm equation or the system of linear Fredholm equations of the first kind, depending on the problem studied, i.e. heat conduction or elasticity problem, respectively. For a binary representation of 'replacement' microstructure, being a two-phase statistically isotropic medium, the computational micromechanics framework is used. The latter represents any microstructure morphology that can be obtained using the representation of a two-dimensional digital image composed of $n \times n$ pixels. For this case the considerations are limited to the problem of heat conduction in a two-phase medium. Simulated annealing algorithm is used to solve the inverse problem. The correctness and effectiveness of the methodology proposed is illustrated by a sequence of numerical examples.